A Literature Review and Comparative Analysis on Radial Basis Function Networks

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*Abstract*—Radial Basis Function Networks (RBFNs) are powerful tools used for universal approximation, which make them applicable to an abundance of fields and applications. They also exhibit a faster learning speed than many of their counterparts, making them a lucrative algorithm. The overall architecture and composition of an RBFN are briefly discussed, before three different ways in which RBFNs have been applied with real-world data are investigated. The diverse methods in which these networks can be adopted will also be analyzed, through the comparison between two models per application.

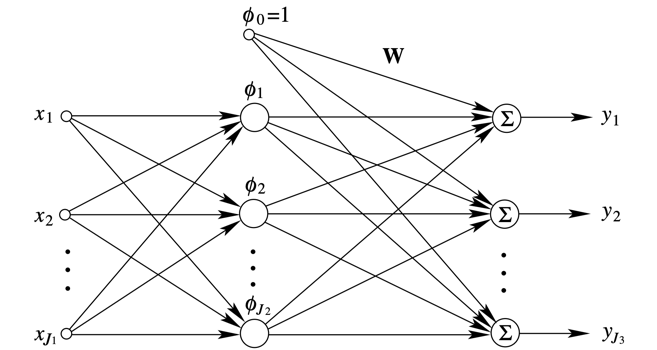
Keywords—Radial Basis Function Networks; Hidden Layer; Euclidean Distance; Passenger Flow; Epileptic Seizure; Flood Risk

# Introduction

Radial Basis Function Networks (RBFNs) were first developed in 1988 by Broomhead and Lowe [1]. RBFNs are feedforward neural networks used in classification and function approximation, and demonstrate some versatility, as both supervised or unsupervised learning may be applied to the network.

This neural network’s architecture comprises three distinct layers: the input layer, the hidden layer, and the output layer. The input layer is a collection of *n* nodes, that receive the data of an *n*-dimensional vector, *x*. These nodes are then fully connected to the neurons of the hidden layer, each utilizing a radial basis function. The RBFs acts as non-linear activation functions. Finally, the hidden layer is also fully connected to the output layer, which intakes the output of every RBF neuron, multiplied by some weight, and sums them to produce a final output scalar. Multifarious radial basis functions can be used, such as linear, thin-plate spine, and logistic functions however, the most prevalently used RBF is the Gaussian function.

A Learning can take place within two aspects of the radial basis function network, with regards to RBF neuron centers and weights, both of which playing a crucial role in the performance of the network. This takes place via the minimization of an error criterion, typically mean square error



1. Architecture of an RBFN [2]

(MSE). The centers can be selected either as a random subset of the training set, in which case no learning occurs, or through the use of a clustering algorithm, such as k-Means Clustering. The distance from the centers can then be measured by numerous measures, most popularly Euclidean distance, but also Mahalanobis distance, Manhattan distance, etc.

As for the weights, they are learned through a linear optimization problem, which are solvable by algorithms such as least-squares (LS) or gradient descent [2]. Naturally, there are additional extensive techniques that are utilized in the learning process, with respect to both supervised and unsupervised approaches, that would be too broad to discuss.

In this paper, various applications of the RBFN from literature will be discussed and examined. RBFNs have been employed in a plethora of fields, however the below review will be limited to the scopes of: Short-term Passenger Flow, Epileptic Seizure Detection, and Flood Detection. Susequent to the literature review, the models built in the aforementioned studies will be compared and analyzed, with respect to the hyperparameters used, the accuracy of the network and any additional algorithms employed.

# Literature Review for RBFNs

## Application 1: Short-term Passenger Flow

#### Paper 1: Short-term passenger flow prediction under passenger flow control using a dynamic radial basis function network [3]

A dynamic radial basis function network was built to predict when and where potential spikes of passenger flow in public subway transport may occur.

#### Paper 2: Forecasting short-term subway passenger flow under special events scenarios using multiscale radial basis function networks [4]

A multiscale radial basis function network (MSRBF) is proposed that also forecasts increases in passenger flow, for the aims of crowd control and the mitigation of congestion.

## Application 2: Epileptic Seizure Detection

#### Paper 1: Epileptic seizure detection in EEG signals using sparse multiscale radial basis function networks and the Fisher vector approach [5]

A multiscale radial basis function network is constructed to identify the occurrence of an epileptic seizure in an electroencephalography (EEG) signal.

#### Paper 2: Epileptic seizure classification of EEGs using time–frequency analysis based multiscale radial basis functions [6]

A so-called ‘MRBF-MPSO-SVM’ (multiscale radial basis function – modified particle swarm optimization – support vector machine) classification approach is presented to distinguish between seizure and seizure-free EEG signals.

## Application 3: Flood Detection

#### Paper 1: A machinelLearning ensemble approach based on random forest and radial basis function neural network for risk evaluation of regional flood disaster: A case study of the Yangtze River Delta, China [7]

A random forest was implemented to determine important flood-risk metrics, and the risk of a flood occurring was then assessed through a radial basis function neural network.

#### Paper 2: Satellite image classification using genetic algorithm trained radial basis function neural network, application to the detection of flooded areas [8]

A genetic algorithm was used to train a radial basis function network that classifies satellite images for the recognition of flood areas.

# Comparative Analysis

## Short-term Passenger Flow

In both neural networks, a gaussian radial basis function was used, and the Euclidean norm computed. The models were built on data from the Beijing metro, but the specific stations and dates used vary. Each used the data of 3 stations, however only one station’s data was used in common (WKS station). Furthermore, the data in the first study was aggregated to every 30 minutes, whereas in the other 15-minute intervals were used.

In the first network (RBF), the generalized cross-validation (GCV) and error reduction coefficient (ERR) were used to select the final inputs. Fuzzy c-Means Clustering (FCM) was used to find the centers, and Orthogonal Least Squares (OLS) to determine the weights. With regards to model 2 (MSRBF), fast FCM with partition index was used to learn the cluster centers.

Both models evaluated their performance based on three metrics: MAPE (mean absolute percentage error), VAPE (variance of absolute percentage error), and RMSE (root mean square error).

For the sake of comparison, only the predictions made by each model for the WKS station will be examined. It is important to note that, even though the data is acquired from the same subway station, the first model uses data samples of March 3rd, 2015, and the other July 21, 2012.

“Fig. 2” shows the MAPE, VAPE and RMSE of both models with respect to WKZ station. It can clearly be denoted that the multiscale radial basis function network implemented in the second study outperforms the dynamic RBF.

1. Short-term Passenger Flow RBF vs MSRBF

| Model | Performance Metric | | |
| --- | --- | --- | --- |
| MAPE | VAPE | RMSE |
| RBF (paper 1) | 8.7656 | 0.4357 | 112.9729 |
| MSRBF (paper 2) | 11.8038 | 1.5910 | 62.7863 |

1. Values of three performance metrics (MAPE, VAPE, RMSE) for short-term passenger flow networks (RBF, MSRBF)

## Epileptic Siezure Detection

Both studies use the dataset of EEGs provided by Bonn University, but the first model employed an addition dataset recorded by Neurology and Sleep Center Hauz Khas, New Delhi.

The two presented multiscale radial basis functions, which comprise a set of multiple RBFs, each with multiple scale parameters (kernel widths). They used Gaussian kernels and used the Euclidean norm to measure distance. Parameter approximation was done by modified particle swarm optimization (MPSO). The models then fed into a support vector machine (SVM) as their final stage of classification.

Where they differ is that in the first paper (MRBF1), OLS was used to exclude any redundant or irrelevant features, and GCV to ascertain the reasonable model size. The other model (MBRF2) instead extracted relevant features through the use of time-varying autoregressive modelling (TVAR).

Since both models measured their behavior using accuracy (ACC), sensitivity (SEN) and specificity (SPE), their performance can be compared with reference to the common dataset (Bonn University). The performance of model 1 on the Hauz Khas dataset will be neglected, as well as the additional performance measure they used (AUC). “Fig. 3” shows these values.

It can be seen that the first MSRBF model proposed exceeds the performance of its counterpart, as it has greater accuracy, sensitivity and specificity.

## Flood Detection

Both papers target data derived from the Yangtze River area in China, a region prone to urban flooding. However, the data used differs between the models, as the first network uses a set of flood-risk indexes (such as rainfall, water area ratio, population density, etc.), whereas the other network processes satellite images of the region.

The kernel function of the RBFN used for both was the Gaussian function, as well as Euclidean distance for fitting.

In model 1, a minimal value of the loss function was found when a learning rate of 0.005 and penalty coefficient of 1 were used. The initial accuracy was 0.55 after 10000 epochs. However, when a random forest was utilized to preprocess the data and select the necessary features (by excluding any indexed with an influence factor of less than 10%), the accuracy improved to 0.62 after only 3000 iterations.

For the second model, the RBFN was trained using a genetic algorithm (based on a 4-step iterative process: population initialization, fitness computation, selection, and

1. Epileptic Seizure Detection MRBF1 vs MRBF2

| Model | | Performance Metric | | |
| --- | --- | --- | --- | --- |
| ACC | SEN | SPE |
| MRBF1 (paper 1) | SF | 99.30 | 99.34 | 99.27 |
| SZ | 100 | 100 | 100 |
| MRBF2 (paper 2) | SF | 97.60 | 96.80 | 97.20 |
| SZ | 100 | 100 | 100 |

1. Values of three performance metrics (ACC, SEN, SPE) for epileptic seizure detection networks of two subsets (SF, SZ)

mutation).

The values of the hyperparameters were not specified. The overall accuracy of this network was 94.92% and 96.09% for pre- and post-flooding images respectively.

# Conclusion

Radial basis function networks have been used impactfully across a vast scope of applications, including, but not limited to, passenger flow prediction, epileptic seizure detection from EEG signals, and flood risk detection and classification. Within the realms of each field, the way in which RBFNs can be utilized is manifold, and the effectiveness and practicality of the model can vary based on multiple criteria. It has been seen that RBFNs can play a monumental role in many major aspects of life, from healthcare to disaster prevention. For this reason, it is imperative that the research and exertion into building models over a range of applications never ceases. By studying ways in which the network has already been applied, novel networks can be proposed that build upon and enhance the performance of previous work.

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